

Solving Quadratics with the Quadratic Formula

- ① Make sure equation = 0
- ② Find a, b, and c
- ③ Plug in values
- ④ Simplify discriminant
- ⑤ Finish simplifying

Quadratic Formula The Quadratic Formula can be used to solve any quadratic equation once it is written in the form $ax^2 + bx + c = 0$.

Quadratic Formula	The solutions of $ax^2 + bx + c = 0$, with $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
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Example

Solve $x^2 - 5x = 14$ by using the Quadratic Formula.

- ① Rewrite the equation as $x^2 - 5x - 14 = 0$
- ② $a = 1$ $b = -5$ $c = -14$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

③
$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$$

Replace a with 1, b with -5, and c with -14.

$$b^2 - 4ac = 81$$

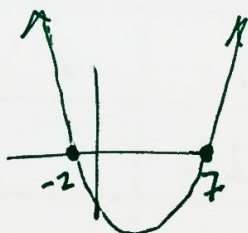
$$= \frac{5 \pm \sqrt{81}}{2}$$

$$= \frac{5 \pm 9}{2}$$

$$= \text{or}$$

$$\begin{aligned} \frac{5+9}{2} &= \frac{14}{2} = 7 \\ \frac{5-9}{2} &= \frac{-4}{2} = -2 \end{aligned}$$

Simplify.



The solutions are -2 and 7.

Roots and the Discriminant

Discriminant	The expression under the radical sign, $b^2 - 4ac$, in the Quadratic Formula is called the discriminant .
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Roots of a Quadratic Equation

Discriminant	Type and Number of Roots
$b^2 - 4ac > 0$ perfect square (positive)	2 real solutions → rational
$b^2 - 4ac > 0$ not a perfect square	2 real solutions → irrational
$b^2 - 4ac = 0$	1 real solution
$b^2 - 4ac < 0$ (negative)	2 complex solutions (i)

Example

Find the value of the discriminant for each equation. Then describe the number and types of roots for the equation.

a. $2x^2 + 5x + 3$

The discriminant is

$$b^2 - 4ac = 5^2 - 4(2)(3) \text{ or } 1.$$

The discriminant is a perfect square, so the equation has 2 rational roots.

b. $3x^2 - 2x + 5$

The discriminant is

$$b^2 - 4ac = (-2)^2 - 4(3)(5) = -56$$

The discriminant is neg, so the

equation has 2 complex solutions

$$x = \frac{-(-2) \pm \sqrt{-56}}{2(3)} = \frac{2 \pm i\sqrt{56}}{6} = \frac{2 \pm 2i\sqrt{14}}{6} = \frac{1 \pm i\sqrt{14}}{3}$$

$$\begin{aligned} \sqrt{56} &= \sqrt{4 \cdot 14} \\ &= 2\sqrt{14} \end{aligned}$$

Find $b^2 - 4ac$, then check the appropriate column for each equation.Complex
↓

	$b^2 - 4ac$	one double rat. root	two different rat. roots <i>rational</i>	two different irrat. roots <i>irrational</i>	no real roots
1. $x^2 + x - 2 = 0$	9		✓		
2. $\overset{a}{3}x^2 - \overset{b}{22}x + \overset{c}{7} = 0$ $(-22)^2 - 4(3)(7)$	400		✓		
3. $x^2 - 8x + 4 = 0$	48			✓	
4. $2x^2 - 9x + 13 = 0$	-23				✓
5. $4x^2 - 12x - 39 = 0$	768			✓	
6. $4x^2 - 12x + 9 = 0$	0	✓			
7. $4x^2 - 20x + 5 = 0$	320			✓	
8. $15x^2 + 11x - 12 = 0$	841		✓		
9. $5x^2 - 7x + 3 = 0$	-11				✓
10. $6x^2 - 11x - 21 = 0$	625		✓		
11. $4x^2 + 20x + 25 = 0$	0	✓			
12. $9x^2 + 30x + 25 = 0$	0	✓			
13. $2x^2 + 6x - 9 = 0$	108			✓	
14. $4x^2 - 12x - 19 = 0$	448			✓	
15. $6x^2 + 10x - 5 = 0$	220			✓	
16. $16x^2 - 8x + 14 = 0$	-832				✓

6-5

Skills Practice

The Quadratic Formula and the Discriminant

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

1. $x^2 - 8x + 16 = 0$

a. 0 b. 1 double rational root c. $x = 4$
1 real solution

2. $x^2 - 11x - 26 = 0$

3. $3x^2 - 2x = 0$

4. $20x^2 + 7x - 3 = 0$

5. $5x^2 - 6 = 0$

6. $x^2 - 6 = 0$

7. $x^2 + 8x + 13 = 0$

8. $5x^2 - x - 1 = 0$

9. $x^2 - 2x - 17 = 0$

10. $x^2 + 49 = 0$

11. $x^2 - x + 1 = 0$

12. $2x^2 - 3x = -2$

Solve each equation by using the method of your choice. Find exact solutions.

13. $x^2 = 64$

14. $x^2 - 30 = 0$

15. $x^2 - x = 30$

16. $16x^2 - 24x - 27 = 0$

17. $x^2 - 4x - 11 = 0$

18. $x^2 - 8x - 17 = 0$

19. $x^2 + 25 = 0$

20. $3x^2 + 36 = 0$

21. $2x^2 + 10x + 11 = 0$

22. $2x^2 - 7x + 4 = 0$

23. $8x^2 + 1 = 4x$

24. $2x^2 + 2x + 3 = 0$

25. **PARACHUTING** Ignoring wind resistance, the distance $d(t)$ in feet that a parachutist falls in t seconds can be estimated using the formula $d(t) = 16t^2$. If a parachutist jumps from an airplane and falls for 1100 feet before opening her parachute, how many seconds pass before she opens the parachute?